Pricing Securities When Markets Are Incomplete Via OLG Modeling With Aggregate Risk

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Long-Standing Question

Major Answer to Date -

Arbitrage Pricing

Arbitrage Pricing

- Find Marketed Assets Whose Returns Span Return on Unpriced Security

- Price Unpriced Security Based on Prices of Spanning Marketed Assets

Concerns With APT

- Finding Market Assets that Span Unpriced Asset X May Be Very Difficult
- Arbitrary Assumptions About Process Relating Return on X to Return on Market Assets
- Implementation Difficulty When X's Return Depends on Multiple Lagged Returns to Market Assets
- Non-Linearity In Return Process

GE-Based Asset Pricing

- Permits Determining the Value of All Securities
- No Need To Find Spanning Assets
- Pricing of X Based On Compensating Differentials in Expected Remaining Lifetime Utility
- Pricing Based on Simple Question: How Much Would Agent Give Up Today for Claim to Future X?

GE-Based Asset Pricing For Unborn

- Future Generations Can Be Modeled As Alive, But With No Utility Until They Appear
- Symmetric Treatment With Living Generations Who Have No Utility Once They Disappear
- Related Simple Question. How Much Would Future Generations Pay When They Appear to Receive X?

Life-Cycle Framework

- Features Naturally Incomplete Markets
- Can Now Easily Be Simulated With Aggregate Shocks and Multiple Periods
- Unborn Can’t Implicitly Trade Through Their Living Relatives

Our Model

- 10-period Overlapping Generations Model
- Isoelastic Preferences, Risk Aversion of 2
- Consumption and Portfolio Allocation Choices
- Inelastic Labor Supply, Retirement at Age 8
- Cobb-Douglas Production, Capital's Share is 1/3
- Shocks: AR1 TFP shock, Z_t, and Depreciation
- $\ln(Z_{t+1}) = \rho \ln(Z_t) + \epsilon_{t+1}$, where $\epsilon_{t+1} \sim \mathrm{N}(0, \sigma^2)$
- One Period Safe Bond Market

Calibration

- TFP Calibrated As In Hansen (85), Prescott (86),…
- Depreciation Shocks Calibrated To Reproduce the Variability of the Return on U.S. Aggregate Wealth

	Mean Return to Capital (%)	**S.D. of Return to Capital (%)**
	Model	
No Social Security	5.311	4.628
Social Security	7.671	4.211
	Return to U.S. Wealth Data	
	6.512	4.886

Household Expected Utility Maximization

$$\max E_t[u(c_{1,t}) + \beta u(c_{2,t+1}) + \cdots + \beta^9 u(c_{10,t+9})]$$

s.t.

$$c_{a,t} = L_a w_t + \left[\alpha_{a-1,t-1} \times (1 + \bar{r}_{t-1}) + \left(1 - \alpha_{a-1,t-1}\right) \times (1 + \tilde{r}_t)\right] A_{a-1,t-1} - A_{a,t}$$

where

$c_{a,t}$ is consumption by agent age a at time t and

$\alpha_{a,t}$ is the share of assets allocated to bonds at time t

$\tilde{r}_t$ is the return to capital, $\bar{r}_t$ is the risk-free rate

Solution Method

- Hasanhodzic and Kotlikoff (2013, revised 2017) Solve 80-Period OLG Model with Aggregate Risk
- Based off Marcet (1988) and Judd, Maliar,& Maliar (2009, 2011)
- Trick is to Consider States in Ergodic Space
- Draw path of shocks, guess decision functions as polynomials of state vector, run economy forward, update polynomials, continue till convergence

Valuing Unpriced Securities for the Living

$$EU_{a,t}$$
$$= u(c_{a,t} - m) + \beta E_t[u(c_{a+1,t+1})] + \cdots$$
$$+ \beta^{\tau} E_t[u(c_{a+\tau,t+\tau}) + \tilde{\epsilon}_{t+\tau} \times \bar{X}_{t+\tau}] + \cdots$$
$$+ \beta^{10-a} E_{t+a}[u(c_{10,t+10-a})]$$

$$\frac{dm}{d\bar{X}_{t+\tau}} = \beta^{\tau} \frac{E_t[u'(c_{a+\tau,t+\tau}) \times \tilde{\epsilon}_{t+\tau}]}{u'(c_{a,t})}$$

If $\tilde{\epsilon}_{t+\tau} = 1 \; \forall \; \tau, X$ is a safe asset, otherwise it’s risky

Example – Pricing a Safe Asset for the Living

Initial State: Low Risk-Free Rate, High Return on Capital

	Periods Till Payoff Received								
Current Age	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0.043	0.045	0.045	0.046	0.047	0.047	0.048	0.048	0.049
2	0.043	0.045	0.045	0.046	0.047	0.048	0.048	0.049	
3	0.043	0.045	0.046	0.046	0.047	0.048	0.048		
4	0.043	0.044	0.046	0.046	0.047	0.048			
5	0.043	0.045	0.046	0.047	0.047				
6	0.043	0.045	0.046	0.046					
7	0.041	0.044	0.045						
8	0.041	0.043							
9	0.041								

	Risk-Free Rate	Return on Capital
Current	**0.041**	**0.081**
Average	**0.047**	**0.053**

Example – Pricing a Safe Asset for the Living

Initial State: High Risk-Free Rate, Low Return on Capital

Current Age	Periods Till Payoff Received: 1	2	3	4	5	6	7	8	9
1	0.069	0.067	0.066	0.064	0.063	0.062	0.061	0.060	0.059
2	0.069	0.067	0.066	0.064	0.063	0.061	0.060	0.059	
3	0.069	0.067	0.065	0.064	0.062	0.061	0.060		
4	0.072	0.070	0.068	0.066	0.065	0.063			
5	0.067	0.065	0.064	0.062	0.061				
6	0.070	0.068	0.067	0.065					
7	0.070	0.068	0.067						
8	0.071	0.070							
9	0.072								

	Risk-Free Rate	Return on Capital
Current	0.071	0.026
Average	0.047	0.053

Example – Pricing a Safe Asset for the Living

Initial State: Low Risk-Free Rate, High Return on Capital

Model with Social Security

		Periods Till Payoff Received								
		1	2	3	4	5	6	7	8	9
Current Age	1	0.050	0.053	0.056	0.058	0.060	0.061	0.063	0.064	0.065
	2	0.049	0.052	0.055	0.057	0.060	0.061	0.063	0.064	
	3	0.051	0.053	0.056	0.058	0.061	0.062	0.064		
	4	0.048	0.052	0.055	0.057	0.060	0.062			
	5	0.049	0.052	0.055	0.058	0.060				
	6	0.050	0.053	0.056	0.058					
	7	0.050	0.053	0.056						
	8	0.051	0.054							
	9	0.049								

	Risk-Free Rate	Return on Capital
Current	0.050	0.108
Average	0.069	0.077

Example – Pricing a Safe Asset for the Living

Initial State: High Risk-Free Rate, Low Return on Capital

Model with Social Security

Current Age	Periods Till Payoff Received: 1	2	3	4	5	6	7	8	9
1	0.087	0.087	0.086	0.085	0.084	0.084	0.083	0.082	0.082
2	0.087	0.086	0.086	0.085	0.084	0.083	0.082	0.082	
3	0.085	0.084	0.084	0.082	0.082	0.081	0.080		
4	0.087	0.087	0.086	0.085	0.085	0.084			
5	0.088	0.087	0.087	0.086	0.085				
6	0.088	0.087	0.087	0.086					
7	0.089	0.088	0.088						
8	0.088	0.088							
9	0.087								

	Risk-Free Rate	Return on Capital
Current	0.087	0.041
Average	0.069	0.077

Pricing With Safe vs. Risky Assets

- Difference between prices of safe and risky assets:

$$\frac{E[u'(c_{t+\tau}) \times \epsilon_{t+\tau}]}{u'(c_t)} = \frac{E[u'(c_{t+\tau})]}{u'(c_t)} + \frac{Cov(u'(c_{t+\tau}), \epsilon_{t+\tau})}{u'(c_t)}$$

where $E[\epsilon_{t+\tau}] = 1$

Example – Pricing a Risky Asset for the Living

Initial State: High Risk-Free Rate, Low Return on Capital

Model with Social Security

	Periods Till Payoff Received								
Current Age	1	2	3	4	5	6	7	8	9
1	0.088	0.088	0.087	0.086	0.086	0.085	0.084	0.083	0.083
2	0.087	0.088	0.087	0.086	0.085	0.084	0.083	0.083	
3	0.085	0.086	0.085	0.084	0.083	0.082	0.082		
4	0.088	0.088	0.088	0.087	0.086	0.085			
5	0.088	0.088	0.088	0.087	0.086				
6	0.088	0.089	0.088	0.087					
7	0.089	0.089	0.089						
8	0.088	0.089							
9	0.087								

	Risk-Free Rate	Return on Capital
Current	0.087	0.041
Average	0.069	0.077

Example – Consumption, Risky Asset Covariance

Initial State: High Risk-Free Rate, Low Return on Capital

Model with Social Security

$$\frac{Cov\left(u'(c_{t+\tau}),\epsilon_{t+\tau}\right)}{u'(c_t)} \text{ as a fraction of } \frac{E[u'(c_{t+\tau})]}{u'(c_t)}$$

	Periods Till Benefit Received								
Current Age	1	2	3	4	5	6	7	8	9
1	-0.002	-0.015	-0.024	-0.030	-0.034	-0.037	-0.037	-0.038	-0.040
2	-0.002	-0.014	-0.023	-0.030	-0.034	-0.037	-0.038	-0.039	
3	-0.002	-0.015	-0.025	-0.032	-0.037	-0.040	-0.042		
4	-0.002	-0.014	-0.023	-0.028	-0.032	-0.035			
5	-0.002	-0.014	-0.021	-0.027	-0.032				
6	-0.001	-0.013	-0.021	-0.027					
7	-0.001	-0.012	-0.020						
8	-0.001	-0.013							
9	-0.001								

Example – Pricing a Risky Asset for the Living

Initial State: High Risk-Free Rate, Low Return on Capital

Model with Social Security

Higher Payoff Volatility

		Periods Till Payoff Received								
		1	2	3	4	5	6	7	8	9
Current Age	1	0.091	0.101	0.102	0.101	0.100	0.098	0.095	0.093	0.092
	2	0.090	0.100	0.102	0.101	0.099	0.097	0.095	0.093	
	3	0.088	0.099	0.101	0.100	0.099	0.097	0.095		
	4	0.090	0.101	0.102	0.100	0.099	0.097			
	5	0.090	0.101	0.101	0.100	0.099				
	6	0.090	0.100	0.101	0.100					
	7	0.090	0.101	0.102						
	8	0.090	0.100							
	9	0.089								

	Risk-Free Rate	Return on Capital
Current	0.087	0.041
Average	0.069	0.077

Valuing Unpriced Securities for the Unborn

$$EU_{-a,t}$$
$$= \beta^{a} E_{t+a}\left[u\left(c_{1,t+a} - m\Pi_{i=t+1}^{t+a}(1+s_i)\right)\right]$$
$$+ \beta^{a+1} E_{t+a}\left[u\left(c_{2,t+a+1}\right)\right] + \cdots$$
$$+ \beta^{\tau} E_{t+a}\left[u\left(c_{\tau-a,t+\tau}\right) + \tilde{\epsilon}_{t+\tau} \times \bar{X}_{t+\tau}\right] + \cdots$$
$$+ \beta^{a+9} E_{t+a}\left[u\left(c_{10,t+a+9}\right)\right]$$

$$\frac{dm}{d\bar{X}_{t+\tau}} = \beta^{\tau-a} \frac{E_{t+a}\left[u'\left(c_{\tau-a,t+\tau}\right) \times \tilde{\epsilon}_{t+\tau}\right]}{E_{t+a}\left[u'\left(c_{1,t+a}\right)\Pi_{i=t+1}^{t+a}(1+s_i)\right]}$$

$s_t = \bar{r}_t$ or $\tilde{r}_t$ (safe or risky compensation)

Example – Pricing a Risky Asset for the Unborn

Initial State: High Risk-Free Rate, Low Return on Capital

Model with Social Security

Risky Method of Compensation

	Age at the Receipt of the Payoff								
Current Age	1	2	3	4	5	6	7	8	9
-1	0.089	0.088	0.086	0.086	0.085	0.084	0.083	0.082	0.082
-2	0.089	0.087	0.086	0.085	0.084	0.083	0.082	0.082	0.081
-3	0.087	0.086	0.085	0.084	0.083	0.082	0.082	0.081	0.081
-4	0.087	0.085	0.084	0.083	0.083	0.082	0.081	0.081	0.081
-5	0.086	0.084	0.084	0.083	0.082	0.082	0.081	0.081	0.080
					…				
-18	0.080	0.080	0.079	0.079	0.079	0.079	0.079	0.078	0.078
-19	0.080	0.079	0.079	0.079	0.079	0.079	0.079	0.078	0.078
-20	0.079	0.079	0.079	0.079	0.079	0.079	0.078	0.078	0.078

	Risk-Free Rate	Return on Capital
Current	0.087	0.041
Average	0.069	0.077

Example – Pricing a Risky Asset for the Unborn

Initial State: High Risk-Free Rate, Low Return on Capital

Model with Social Security

Safe Method of Compensation

	Age at the Receipt of the Payoff								
Current Age	1	2	3	4	5	6	7	8	9
-1	0.087	0.086	0.085	0.084	0.084	0.083	0.082	0.082	0.081
-2	0.086	0.085	0.084	0.083	0.082	0.082	0.081	0.081	0.081
-3	0.085	0.084	0.083	0.082	0.082	0.081	0.081	0.080	0.080
-4	0.084	0.083	0.082	0.082	0.081	0.081	0.080	0.080	0.080
-5	0.083	0.082	0.082	0.081	0.081	0.080	0.080	0.079	0.079
					…				
-18	0.077	0.077	0.076	0.076	0.076	0.076	0.076	0.076	0.076
-19	0.077	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076
-20	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076	0.076

	Risk-Free Rate	Return on Capital
Current	0.087	0.041
Average	0.069	0.077

Conclusion

- Near-term safe assets valued at prevailing safe rate
- Long-term safe assets tend to be valued at prevailing mean safe rate
- “Risky assets”, e.g., Social Security benefits, may not be as risky as presupposed
- Valuing unpriced securities for future generations depends on method of compensation

Implications for Fiscal Sustainability Accounting

- Realized Fiscal Gap Equals Zero Along Each Economic Path
- Expected Value of Realized Fiscal Gaps Equals Zero
- With Uncertainty Form the Expected Fiscal Gap
- Fiscal Gap Is a Partial Equilibrium Measure
- Best to Specify Process By Which Fiscal Gaps Are Resolved and Simulate in GE the Range of Outcomes Facing Future Generations
- This is Modern Generational Accounting

Implications for Fiscal Sustainability Accounting

- Literature On Valuing Uncertain Social Security Focused On Closed Group Liability
- Our Findings Support Blocker, et. al. (08) Over Geanakopolos and Zeldes (10)
- Given Arbitrary Nature of Benefit Valuation For Unborn, Open Group Liability Does Not Appear To Be Uniquely Defined
- No Clear Relationship Between Expected Fiscal Gap and Open Group Liability

Thank You!