

Valuing Government Obligations When Markets Are Incomplete

Jasmina Hasanhodzic

Laurence Kotlikoff

Boston University

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Valuing Government Obligations

- Governments promise to make sure or risky payments to current and future generations (e.g., Social Security, medical benefits, etc.)
- Valuing them crucial to assess fiscal sustainability:
Fiscal gap = **discounted present value**
of projected spending – tax receipts
- Intuitively, this value may depend on the size, timing, reliability of promises, age of the recipient agent

Discounting Government Promises

- If markets are complete, use security prices (easy)
- But how to discount when markets are incomplete?
 - Assets don't span long-term states, contingencies
 - Social discount rate no good: feasibility vs. ethics
 - Using fixed interest rate common, but appropriate?

Our Work

- Simulate a multi-period GE OLG model
- Find immediate payments agents require to forego promised government payments => discount rates

– Treat unborn as alive with no utility before birth

E.g., expected utility of agent born in 20 years:

$$\beta^{20} E_t[u(c_1(t+20))] + \beta^{21} E[u(c_2(t+21))] + \dots$$

Do the Promises Include GE Effects?

- Need to specify precisely what the government is promising
- We are generating results with and without promises of general equilibrium impacts

Main Results

- Discount rates depend on state of the economy, age of the agent, size of promises
- Fiscal gap based on GE and PE promises can differ
- Discounting safe PE promises far in future with prevailing safe return may be OK

The Model

- 10-period overlapping generations model
 - Isoelastic preferences, risk aversion of 2
 - Inelastic labor: 1 for workers, 0 for retirees
 - Cobb-Douglas technology with realistic shocks
 - Bond market: one-period safe bonds

Solving OLG Model with Aggregate Shocks

- **Bottleneck**: Dimensionality curse due to many state variables
- **Prior solution methods**
 - Log-linearization (Rios-Rull, '94)
 - Krusell-Smith in OLG (Gourinchas, '00; Storeletten et al. '01)
 - Smolyak (Krueger and Kubler, '06)
- We use Hasanhodzic and Kotlikoff ('13), which builds on Judd, Maliar, Maliar ('09, '11)

Calculating Discount Rates in GE

- Solve the model without any government policy
- Solve the model assuming at time 0 government promises to make God-financed (sure or risky) payments to retirees
- For each agent, find how much the government needs to increase his assets to compensate him for not carrying out the policy => **implied discount rate**
- The sum total of compensations is the **fiscal gap**

Present Value of Promises in GE

- For an agent age $a > 0$ at time 0 when policy is evaluated:

$$F_a = \Delta \text{ Expected Lifetime Utility} / U'(C_a(0))$$

- For an agent age $a < 0$ at time 0 when policy is evaluated:

$$F_a = \Delta \text{ Expected Lifetime Utility} / \beta^a E U'(C_1(-a)) \pi_s (1+r_s)$$

for $s = 0, \dots, -a$ and r_s the random return on capital

Annual GE Discount Rates (in Percent)

| Age | Benefit Size | | | |
|----------|----------------|-------|-------|------|
| | Medium | | Small | |
| | State | | | |
| | Good | Bad | Good | Bad |
| 1 | 3.59 | 4.62 | 3.52 | 4.10 |
| 2 | 3.15 | 3.80 | 3.29 | 3.64 |
| 3 | 2.82 | 3.40 | 2.68 | 3.38 |
| 4 | 2.56 | 3.06 | 2.46 | 3.16 |
| 5 | 2.30 | 3.05 | 2.24 | 2.98 |
| 6 | 2.34 | 3.07 | 2.02 | 2.86 |
| 7 | 2.39 | 3.87 | 2.15 | 3.05 |
| 8 | 4.13 | 6.37 | 2.58 | 3.68 |
| 9 | 5.30 | 10.37 | 3.02 | 4.27 |
| | Risk-free rate | | | |
| | 3.06 | 4.39 | 3.25 | 4.41 |

Interpretation

- Discount rates **decline for workers**: policy lowers savings, hurts youngest workers most (GE effects)
- **Elderly discount more heavily, especially in bad states**: diminishing marginal utility
- Discount rates **uniformly higher in bad states** (low K):
higher $r \Rightarrow$ consume more in future \Rightarrow extra cons.
not valued much
- Small benefit: discount close to, but not equal to the risk-free rate

Calculating Discount Rates in PE

- μ = amount agent age $a > 0$ willing to give up at time t to get a benefit at time $t+k$:

$$\mu = \beta^k EU'(C_{a+k}(t+k)) / U'(C_a(t))$$

- δ = implied discount rate:

$$\mu = 1 / (1 + \delta)^k$$

Annual PE Discount Rates (in Percent)

Typical State

| | | Periods Till Benefit Received | | | | | | | | |
|-------------|---|-------------------------------|------|------|------|------|------|------|------|------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Current Age | 1 | 3.20 | 3.24 | 3.27 | 3.30 | 3.33 | 3.35 | 3.36 | 3.38 | 3.39 |
| | 2 | 3.23 | 3.25 | 3.28 | 3.31 | 3.33 | 3.34 | 3.37 | 3.38 | |
| | 3 | 3.23 | 3.25 | 3.28 | 3.30 | 3.32 | 3.34 | 3.36 | | |
| | 4 | 3.22 | 3.26 | 3.27 | 3.29 | 3.30 | 3.33 | | | |
| | 5 | 3.22 | 3.25 | 3.26 | 3.27 | 3.29 | | | | |
| | 6 | 3.22 | 3.24 | 3.25 | 3.27 | | | | | |
| | 7 | 3.21 | 3.24 | 3.26 | | | | | | |
| | 8 | 3.22 | 3.25 | | | | | | | |
| | 9 | 3.21 | | | | | | | | |
| | | Risk-Free Rate | | | | | | | | |
| | | 3.21 | | | | | | | | |

Annual PE Discount Rates (in Percent)

Bad State

| | | Periods Till Benefit Received | | | | | | | | |
|-------------|---|-------------------------------|------|------|------|------|------|------|------|------|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Current Age | 1 | 4.39 | 4.21 | 4.13 | 4.04 | 3.97 | 3.90 | 3.84 | 3.78 | 3.75 |
| | 2 | 4.41 | 4.23 | 4.14 | 4.04 | 3.97 | 3.90 | 3.82 | 3.77 | |
| | 3 | 4.52 | 4.31 | 4.19 | 4.08 | 4.01 | 3.91 | 3.84 | | |
| | 4 | 4.57 | 4.35 | 4.21 | 4.11 | 4.01 | 3.92 | | | |
| | 5 | 4.58 | 4.35 | 4.21 | 4.07 | 3.99 | | | | |
| | 6 | 4.54 | 4.35 | 4.10 | 4.00 | | | | | |
| | 7 | 4.76 | 4.35 | 4.14 | | | | | | |
| | 8 | 4.42 | 4.22 | | | | | | | |
| | 9 | 4.27 | | | | | | | | |
| | | Risk-Free Rate | | | | | | | | |
| | | 4.57 | | | | | | | | |

Fiscal Gap (in Percent)

PV of Obligations By Age and In Total as Share of GDP

| | | | Current Age | | | | | | | | | | | Fiscal Gap |
|------------------|-------|----|-------------|-------|-----|------|------|------|-----|------|------|------|------|------------|
| | | | -10 | -9 | ... | -1 | 1 | 2 | ... | 7 | 8 | 9 | 10 | |
| Small Benefit | Bad | GE | -0.01 | -0.01 | ... | 0.13 | 0.40 | 0.65 | ... | 1.99 | 2.23 | 1.65 | 1.04 | 13 |
| | State | PE | -0.02 | -0.03 | ... | 0.14 | 0.31 | 0.42 | ... | 1.79 | 2.40 | 1.82 | 1.04 | 11 |
| | Good | GE | -0.01 | -0.01 | ... | 0.18 | 0.48 | 0.72 | ... | 1.73 | 1.80 | 1.29 | 0.70 | 12 |
| | State | PE | -0.02 | -0.02 | ... | 0.20 | 0.90 | 1.02 | ... | 1.82 | 2.05 | 1.44 | 0.76 | 14 |
| Large Benefit | Bad | GE | -0.07 | -0.08 | ... | 0.24 | 1.15 | 2.09 | ... | 5.87 | 5.40 | 4.54 | 2.71 | 38 |
| | State | PE | -0.13 | -0.16 | ... | 0.26 | 0.80 | 1.15 | ... | 6.82 | 9.74 | 7.56 | 4.44 | 41 |
| | Good | GE | -0.05 | -0.07 | ... | 0.49 | 1.63 | 2.65 | ... | 6.02 | 5.74 | 4.43 | 2.53 | 42 |
| | State | PE | -0.11 | -0.12 | ... | 0.53 | 3.55 | 4.06 | ... | 7.94 | 9.08 | 6.45 | 3.44 | 57 |

Benefits Provides in Periods 8, 9, and 10. Small Benefit is 10 Percent of Average Wage. Large Benefit is 40 Percent of Average Wage.

PV < 0 => implied GE discount rates not well defined

Conclusion

- Building on Judd, Maliar, and Maliar ('09, '11)
can solve large-scale OLG models for the first time
- Discount rates depend on state of the economy, age of the agent, and size and riskiness of gov promises
Fiscal gap based on GE and PE promises can differ
- Governments can use realistic versions of this model to value their obligations

Thank you!